1) Start with a blank, square matrix, listing every 5A team. Each column represents a unique team, going from left to right. Each row also represents each team, going top to bottom. If there are 33 teams, the size of the matrix is 33 x 33 (i.e. 33 rows by 33 columns.) This is the Colley Matrix, called M.

$$[M] = \begin{bmatrix} Ashland & Bend & \dots & & \dots & Woodburn \\ Bend & & \ddots & & & \\ \vdots & & & & & \\ & & & & & \\ \vdots & & & & & \\ Woodburn & & & & & \end{bmatrix}$$

2) To fill in the values of this matrix M, start with the first row of M go column by column, left to right.
    a. If the matrix entry is on the diagonal (i.e. Ashland vs. Ashland, Bend vs. Bend, etc.):
        i. Count the total number of Colley rankable contests (any contest where both opponents are within the same classification) and add 2.
        ii. $[M]_{i,i} = 2 + total\ \#\ of\ Colley\ rankable\ games\ for\ the\ team$
    b. If the matrix entry is not on the diagonal (i.e. Ashland vs. Bend, Ashland vs. Woodburn, etc.):
        i. This entry represents a matchup between the team in the row against the team in the column.
        ii. Count the number of times the teams faced each other in a game, then multiply that count by -1.
        iii. $[M]_{i,j} = -1 \times \#\ of\ games\ against\ that\ opponent$
        iv. Since this matrix M is symmetric, the entry $[M]_{i,j} = [M]_{j,i}$. That means that the values not on the diagonal can be mirrored across the diagonal, flipping the row and column values. (I.e. Ashland vs Bend is the same as Bend vs. Ashland.)

3) Move to the next row and repeat the previous step, counting the contests for all teams.
    a. You can verify the records are accurate by adding up the values in each row. The sum of these row values should be exactly equal to 2. You can also add up the values in each column which should also add up to 2.

4) The Colley ratings method is a solution to a system of linear equations. This matrix M represents the coefficients of the system (i.e. 33 coefficients in the 5A classification.) The solutions to the system are represented by another matrix, b. This matrix b is a vector matrix (i.e. the size of b is 33 x 1; 33 rows in 1 column.) Each row represents the same order of teams as in the rows in matrix M.
   a. The value of each entry in b is calculated from that team's Colley rankable record of wins and losses.
   b. The values in the matrix are the number of wins, minus the number of losses; all divided by 2, then added to 1.
   c. $[b]_i = 1 + \left(\frac{Wins - Losses}{2}\right)$

5) We can now represent the system of linear equations by showing the equation of the system's coefficients multiplied by their variables is equal to their solution.
   a. $[M] \times [x] = [b]$
   b. In this equation, the variables are represented in a similar fashion as the vector matrix b. This variable matrix, called x, is size 33 x 1 with each team having its own value. The entries in this matrix are the teams' Colley ratings.

6) This solution for the variable matrix x is easily solved by taking the inverse of the Colley matrix M cross-multiplied by the solution vector matrix b.
   a. $[M]^{-1} \times [b] = [x]$

The example on the last page is a print out of the 5A Football Colley Matrix. In this example, each team is listed from left to right and top to bottom in the Colley matrix M. The numbers are calculated from the OSAA scores and records database. (These records are known as Contest Teams Scores, or CTS records.)

The "CRR" column is a verification that the CTS records match what the entries in matrix M reflect. The "#" column is the sum of the contest records to ensure that the diagonal values are exactly 2 more than the value in this column (the diagonal values are shaded in gray and underlined in the M matrix.) The "checksum" row is a summation of the values in that column, minus 2. Each values in that row should be zero to help verify the entries are accurate. This row and the extra columns to the right of the last team are not part of the Colley matrix M.

Solving a large system such as this can be very computationally intensive. The inverse of a matrix requires a lot of operations. Some online calculators will take these values and solve for you.

To get a more concrete representation of each row and column in the matrix, the green highlighted row (Hermiston) is a matrix representation of the following equation:

$$(0)X_1 + (0)X_2 + (0)X_3 + (0)X_4 + (0)X_5 + (0)X_6 + (0)X_7 + (0)X_8 + (0)X_9 + (6)X_{10} + (0)X_{11} + (-1)X_{12} + (0)X_{13} + (0)X_{14} + (0)X_{15} + (0)X_{16} + (0)X_{17} + (0)X_{18} + (0)X_{19} + (0)X_{20} + (0)X_{21} + (0)X_{22} + (0)X_{23} + (0)X_{24} + (-1)X_{25} + (0)X_{26} + (0)X_{27} + (-1)X_{28} + (0)X_{29} + (0)X_{30} + (-1)X_{31} + (0)X_{32} + (0)X_{33} = 3$$

Excluding the zeroed out variables, the equation is simplified to:

$$(6)X_{10} + (-1)X_{12} + (-1)X_{25} + (-1)X_{28} + (-1)X_{31} = 3$$

Each team's row represents a separate equation of 33 unknown variables and a solution from that team's record. The solution of all 33 variables (where X_1 through X_{33} are the team's Colley ratings) is a unique solution; there can be only one combination of rating to satisfy all equations. I.e. if the ratings are plugged back into the original equations, then each equation will still be true. For example:

$$(6)(0.782109) + (-1)(0.170355) + (-1)(0.551905) + (-1)(0.779140) + (-1)(0.191251) \cong 3$$